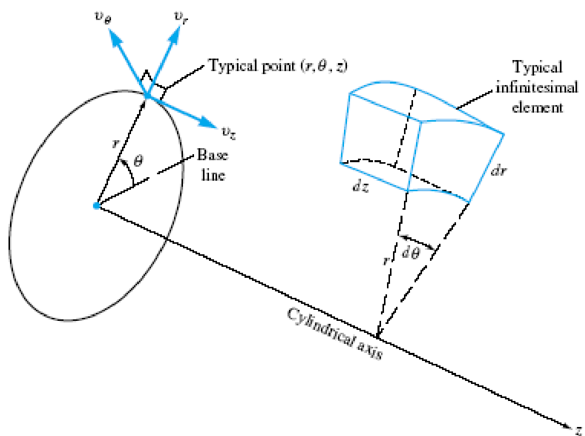
Sahand University of Technology
Chemical Engineering Faculty
Instructor: Dr. Alizadeh

Advanced Fluid Mechanics HW2
Return date: With Midterm exam
Fall Semester

1. By selecting an appropriate element and using mass balance, derive continuity equation for cylindrical coordinate



2. The stream function for a two-dimensional, nonviscouse, incompressible flow field is given by the experssion

$$\psi = -2(x - y)$$

Where the stream function has the units of ft^2/s with x and y in feet.

a) Is the continuity equation satisfied?

b) Is the flow field irrotational? If so, determine the corresponding velocity potential.

c) Determine the pressure gradient in the horizontal x direction at the point x=2 ft, y= 2 ft.

3. A certain flow field is described by this stream function where A and B are positive constants:

$$\psi = A\,\theta + B\,r\sin\theta$$

Determine the corresponding velocity potential and locate any stagnation points in this flow field.

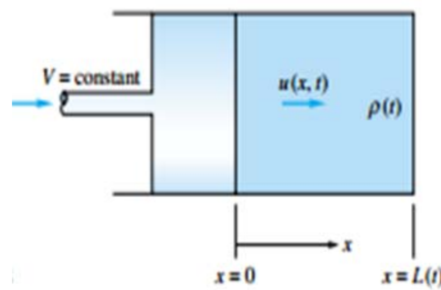
4. A laboratory test tank contains seawater of salinity S and density ρ. Water enters the tank at conditions (S_1, ρ_1, A_1, V_1) and is assumed to mix immediately in the tank. Tank water leaves through an outlet A_2 at velocity V_2. If salt is a 'conservative' property (neither

created nor destroyed), use the Reynolds transport theorem to find an expression for the rate of change of salt mass M_{salt} within the tank.

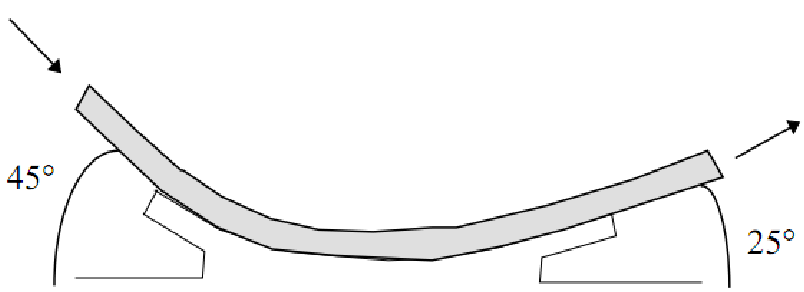
5. Check whether the following velocity relations satisfy the requirements for steady irrotational flow.

   (i) $u = x + y$, $v = x - y$ (ii) $u = xt^2 + 2y$, $v = x^2 - y\, t^2$ (iii) $u = xt^2$, $v = xyt + y^2$

6. A piston compresses gas in a cylinder by moving at constant speed V, as in figure. Let the gas density and length at t=0 be ρ_o L_o, respectively. Let the gas velocity vary linearly from u=V at the piston face to u=0 at x=L. if the gas density only with time, find and expression for ρ(t).



7. The figure below shows a smooth curved vane attached to a rigid foundation. The jet of water, rectangular in section, 75mm wide and 25mm thick, strike the vane with a velocity of 25m/s. Calculate the vertical and horizontal components of the force exerted on the vane and indicate in which direction these components act.



8) The velocity field for a flow is given by:

$$\mathbf{u} = \frac{-Cy}{\sqrt{x^2+y^2}}\,\mathbf{i} + \frac{Cx}{\sqrt{x^2+y^2}}\,\mathbf{j}$$

where *C* is a constant. Transform these Cartesian velocity components into cylindrical velocity components u_r and u_θ . Determine the equations for the streamlines and make a sketch.